

# ***Game Theory Analysis of Algebraic Content Selection in Middle School Mathematics Curricula***

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**Abstract:** The algebra section of middle school math textbooks is indispensable. The characteristics of the textbook and the rationality of its elements need to be algebra content of middle school math textbooks mainly consists of algebraic formulas, equations and inequalities, functions and other elements. This paper focuses on modeling the analysis based on game theory and then exploring the rationality of the algebra textbook study is conducted by reviewing the literature and constructing an analytical model based on cooperative game and Nash equilibrium theory. It is obtained that in the strategy where all three elements are used at the same time, the one that focuses on the orchestration function can maximize the reorganization of unit algebra content design is needed to achieve rational weighting of individual algebra selection of elements needs to fully reflect the articulation between the individual algebra elements. Algebraic elements are designed to spiral in difficulty to match the needs of teachers and students using them.

**Keywords:** middle school algebra, game theory, selection of algebra elements for textbooks

## **1. Introduction**

Nowadays, the junior high school mathematics algebra textbooks have the following characteristics: first, the junior high school mathematics textbooks are written according to the Curriculum Standards. Important mathematical content is written in a spiral progression, and the teaching of algebra in junior high school mathematics is based on the textbook, which helps students to clarify the learning objectives and easily accept the knowledge presented in the textbook, but there is still room for improvement in the complete presentation of the structure of the algebra content of middle school math textbooks mainly presents more intuitive definitions and properties. There is a more patterned approach to solving problems, with less in-depth logical algebraic thinking. This mode of presenting materials has worked well at the middle school level, but is not appropriate for students who want to continue to explore mathematics in theory is the study of the existence of multiple decision makers in the decision-making problem, game theory in mathematics is mainly used as a modeling paradigm for research [1]. In order to improve the effectiveness of using the algebra part of the junior high school mathematics textbook, it needs to screen the key elements of the junior high school mathematics algebra and control of different algebraic elements in the textbook through the corresponding values of the payoff function of the cooperative game allows for optimal use of the textbook.

## 2. Algebra Elements in Middle School Math Textbooks

The algebraic component of the middle school mathematics curriculum is structured into three principal segments: algebraic expressions, the study of equations and inequalities, and an exploration of functions [2]. Algebraic formula is one of the important concepts of math algebra learning. This paper uses the definition of algebraic formula in the first book of the seventh grade of the Humanistic Education Edition. The formula that connects the number and the letter that represents the number with the basic operation symbols (operations including addition, subtraction, multiplication, division, multiplication and prescribing) is called an algebraic [3]. In China, students' systematic knowledge of algebraic formulas mainly originates from middle school. Algebra as a necessary knowledge of mathematical problem solving, can be used directly for mathematical problem solving, but also through the geometry. The geometry of the content of the algebraic solution, can be said to be a very necessary tool. For equations, we know that equations containing unknowns are equations, but this statement is not rigorous enough. Scholar Zhang Dianzhou pointed out that an equation is a kind of equivalence relationship established between an unknown and a known number in order to find [4]. An inequality is a formula that represents an unequal [3].

Equations and inequalities can belong to the same category of "formulas", but there is a slight difference. Equations must contain unknowns, while inequalities do not necessarily contain unknowns. Function is one of the few mathematical areas in junior high school mathematics that gives a clear mathematical definition. Here we use the definition of function in the second book of the eighth grade of the Renmin Jiao Tong edition. In there are two variables  $x$  and  $y$ , and for each determined value of  $x$ ,  $y$  has a unique determined value corresponds to it, then we say that  $x$  is the independent variable, and  $y$  is a function of  $x$  [5-6]. The concept of a function changes in the later stages of math. Since this paper focuses on the middle school level, we use the "numerical" definition of middle school.

## 3. The Game Model Embodied in the Selection of Algebraic Elements in Middle School Mathematics Textbooks

### 3.1. Game Modeling of Algebraic Elements in Mathematics Textbooks

#### 3.1.1. Textbook Algebraic Elements Selection and Game Modeling

Game theory is in a way the embodiment of mathematics, so this paper will use the Nash equilibrium theory of game theory to game analyze the choice of each textbook element and find the most appropriate way to set up the element. The use of game theory in the analysis of algebra elements of junior high school mathematics textbooks is also a convergent practice of attributing game theory, which originates to mathematics [7]. Scholar Camerer has suggested that simple games help establish phenomena that should be included in economics beyond game [8]. Game theory is the economic part of the study of "game rules". It provides a framework for solving design problems, where design involves responsibility for details, which creates the need to deal with complexity, and new tools are needed to deal with complex [9].

The first estimate of the corresponding benefits of the different algebraic elements is based on the percentage of algebraic elements. Algebraic formulas are the prerequisite for learning equations and inequalities, equations and inequalities pave the way for learning functions, and functions are the comprehensive algebraic knowledge used as the key to algebraic learning. Therefore, the gains for algebraic formulas, equations and inequalities, and functions are made to be the gain values of 1, 1, and 2, respectively, as shown in Table 1.

Table 1: Algebraic triad of corresponding gains.

algebraic element	algebraic formula
earnings	1

Let the algebraic equation be A, the equation and inequality be B, and the function be C. The combination of the three elements can produce a total of seven pairings, ABC, AB, AC, BC, A, B, and C. Assuming that A, B, C three elements when one of the elements for the focus of the setup, the element gains double, the other elements gain take half of the original gain, can be obtained as follows in Table 2.

Table 2: Table of returns to different strategies in the math textbook algebra element of the game.

	ABC	AB	AC	BC	A	B	C
Focus A	3.5	2.5	3		2		
Focus B	3.5	2.5		3		2	
Focus C	5		4.5	4.5			4

According to the gains generated by different combinations and focuses in Table 2, it is easy to find that one of the highest gain strategies is the combination of the three elements of ABC together and focusing on the use of C. That is to say, it is necessary to have a combination of the three elements of algebraic formulas, equations and inequalities, and functions, and focus on the use of C in the design and layout of the textbook. Through the construction of the purely theoretical model and the results mentioned above, it provides ideas for the combination of the elements of the elements of middle school mathematics and the content setting of the content.

### 3.1.2. Game modeling about teacher-student use textbook

The main actors in the selection of the elements of the algebra textbook are the teacher and the students. Teachers have two strategies: adequately prepared and inadequately prepared lessons; students also have two strategies: conscientious learning and unconscientious learning. The variables are parameterized as follows:  $\alpha$  denotes the probability that the teacher is adequately prepared for class, and  $\beta$  denotes the probability that the student is serious about learning. The cost to the school of adequate teacher preparation is C, the loss to the school of inadequate teacher preparation and students' learning inattentiveness is B, and the gain to the students of the other scenarios is A.

In addition, the cost of an adequately prepared teacher is D1, the cost of an inadequately prepared teacher is D2, the teacher gains M from accomplishing the task of preparing the lesson, and the students gain a reduction of Q in the classroom of an inadequately prepared teacher as Figure 1 shown.

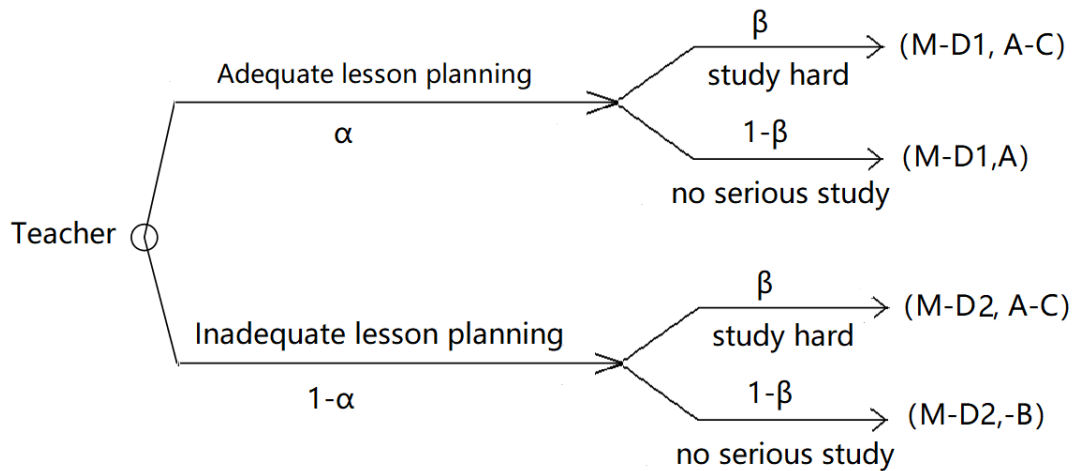


Figure 1: Game model of textbook use.

Teachers who are well-prepared may calculate expectations, equation (1):

$$E_1 = \beta(M - D_1) + (1 - \beta)(M - D_1) = M - D_1 \quad (1)$$

Teachers who are not adequately prepared may calculate expectations, equation (2):

$$E_2 = \beta(M - D_2) + (1 - \beta)(M - D_2) = M - D_2 - \beta Q \quad (2)$$

This can be deduced from the equality of the two expectations:  $\beta = \frac{D_1 - D_2}{Q}$

### 3.2. Analysis of the results of the game model with algebraic elements

In the game of algebra elements in middle school math, the players are students and teachers. Students use the textbook to acquire more algebra-related ideas and build a framework of algebraic knowledge in their minds. Teachers refer to the textbook content and the key points to prepare their lessons and find ways to help students to put algebraic logic in place. Based on the interests as above, it can be seen that the choice of the algebra content of middle school math textbooks is subordinate to the content of the game that can be handled by the Nash equilibrium. Nash equilibrium requires the assumption that the participants are completely rational, and the information of the game is complete, so that if any of the participants change their strategies, it is very likely to affect the combination of [10]. Game theory focuses on what combination of strategies can maximize the benefits of each party. In the game in this paper, the three parties cooperate and focus on the element with the greatest benefit to maximize cooperation, which indicates that the three algebraic elements are interrelated and promote each other, and no one element can be left vacant. Algebraic formulas are responsible for allowing students to express themselves algebraically and to express themselves mathematically by replacing the abstract with the figurative. The study of equations and inequalities enhances the student's approach to solving for unknowns and the skill of framing the range of unknowns. Functions are central to the study of algebra, and the study of functions synthesizes the roles of algebraic formulas and equations and inequalities. The rational combination of the three elements can maximize the effectiveness of the middle school algebra textbook, so the core idea of the textbook is to build a sound algebraic structure. A perfect algebraic structure requires a reasonable arrangement of the elements of algebra design, in order to maximize the mobilization of students' independent learning,

the formation of student-oriented constructive learning, and truly cultivate students' ability to learn algebra.

Of the three elements of algebra, the benefits of focusing on function use are the greatest, illustrating the fact that learning algebra is a spiraling learning process. When a higher level of comprehensive function use surface is reached through the first two required element pathways, the benefits of all the elements complement each other and maximization of gains is reached. This also shows that in middle school algebra learning, simply memorizing and practicing methods of algebraic formulas, solving equations and inequalities cannot achieve the most ideal state of algebraic use, and one should focus on their applicability in real-world scenarios. For middle school math algebra teaching, if the teacher can let the students add more thinking in the simple and repeated practice. If teachers can make students think more in the simple and repeated practice, and turn mechanical practice into rational thinking, master the application and core ideas of algebra, and build a complete framework of algebraic structure, it will be a maximum gain for students. In the future, students will also be able to exercise their ability to learn independently, and this level of independent learning is the highest level of algebra learning.

For the second model, it can be obtained that 1) If the probability of students learning seriously, the expected utility of the teacher who prepares adequately is greater than that of the teacher who prepares insufficiently during lesson preparation, the teacher will tend to choose to prepare the lesson adequately. 2) If the probability of students learning seriously, the expected utility of the teacher who prepares adequately is less than that of the teacher who prepares insufficiently during lesson preparation, the teacher will tend to choose inadequate lesson preparation. 3) If the probability of students learning conscientiously, during lesson planning, the expected utility of a teacher who is well-prepared equals the expected utility of a teacher who is not well-prepared, and it is possible for a teacher to choose to be either well-prepared or not well-prepared.

#### **4. Path optimization for algebraic element selection**

##### **4.1. Reorganization of the algebraic content of the unit**

Each unit in the middle school math textbook has a relatively fixed amount of algebra to learn. The length of time required for different units of study is determined by the different circumstances of different students. Nothing exists in isolation from the world because the world as an organic whole is connected in every way. Math content and the algebra content under which math is divided is systematic and structured and is constantly growing [11]. A textbook with a sound algebraic structure can effectively link algebraic knowledge, making it easier for students to see the connections between the various aspects of algebra and to deepen their understanding of algebra so that they can apply it flexibly.

Because of the algebraic abstraction in mathematics, and the sequence of algebraic knowledge learning also plays an important role in whether or not the algebraic structure can be linked up [12]. Throughout the current mainstream textbooks of junior high school mathematics, it is found that there exist certain units of different algebraic knowledge corresponding to the arrangement of class time still need to be further negotiated. For example, whether to further increase the weight of functions. This is because in game analysis, the learning of function module will have a more obvious promotion of algebraic gains. Students through the textbook reasonable unit algebra content set, can more quickly understand the connection between the knowledge learned, quickly build out the algebraic framework is the main purpose of the unit algebra content reorganization. For students, the ability to subconsciously form a more complete algebraic structure will be very beneficial to future algebra learning. Although the seriousness of student learning cannot be monitored in every class, it can be

monitored in an orderly sampling of selected classes, which also improves the teacher's profitability in completing the preparation M and makes the material effective.

#### **4.2. Upgrading the logical construction of elements**

Algebra textbooks are the main reference for teaching middle school math, and both students and teachers rely on them for learning and teaching. Algebra textbooks include algebraic formulas, equations and inequalities, and functions, which are clear. However, when broken down into smaller theorems, definitions, formulas, will be side branches and cover a wide range of topics. It is especially important to structure the textbook in a logical order so that the many algebraic facts are well organized and logically well thought out. Algebra textbooks, as the main books for learning algebra, are logical and comprehensive, which will make it easier for more learners and help them to progress in their algebra studies [13]. To improve the logical construction of algebra elements, the first step is to adopt a spiral progression of knowledge, which is characterized by increasing difficulty and increasing knowledge in the content. In terms of setting up the algebra content, algebraic formulas will be taken first, equations and inequalities will be added after the foundation of algebraic formulas, and finally the functions module will be taken. Corresponding bedding function content should also be set up in both the algebraic formulas and the equations and inequalities sections in order to pave the way for the function module which is the final focus. The corresponding penalty Q should be increased for teachers who have not prepared the lessons adequately, but a sound assessment mechanism should be established to judge whether the teachers have prepared the lessons adequately so as to be fair and impartial.

#### **4.3. Optimizing the form of algebraic structures**

Having set up a more logical algebraic logic for the textbook in point two, the algebra textbook next needs to have an appropriate form of algebraic structure. Since the audience of middle school algebra textbooks is middle school students in the early stages of puberty, they are growing rapidly physically but more slowly mentally [14]. When students enter middle school, the number of subjects increases and the pressure of learning increases, and they need to adapt to more subject knowledge and learning content in a shorter period. Therefore, in order to be close to the psychological condition of students, algebra teaching materials should not be too theoretical, should be added under the premise of a clear framework of interesting mathematical knowledge, mathematical culture into it, in order to optimize and improve the structure of the form of algebra. Mathematical culture has a lot of content related to the knowledge learned in junior high school, if the mathematical culture can be skillfully integrated into the algebra textbook, it will make the textbook more readable.

### **5. Conclusion**

The philosophy of classroom instruction ought to be centered on the students, fostering their active engagement and initiative. To this end, incorporating collaborative inquiry and practical exercises into the educational materials can significantly enhance the learning experience. Students can have more opportunities to explore learning, so as to achieve the integration of knowledge into practice, to achieve the living application. For students with different algebra fundamentals, the textbook can set up basic topics and extension topics, to clarify the role of the topics. This will allow intermediate students to continue to make progress and advanced students to further develop their skills. Algebra materials should be set up in such a way that teachers and students can "motivate each other" as much as possible, thus effectively reducing the cost of choosing the elements of algebra and building effective algebra materials.



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